Class: XII
SESSION : 2022-2023
SUBJECT: Mathematics SAMPLE
QUESTION PAPER - 18
with SOLUTION

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions :

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

Section A

1. $\int \frac{x^2-1}{x^4+3x^2+1} dx$ is equal to **[1]**

   a) $\tan\left(x+\frac{1}{x}\right)+C$ b) $\tan^{-1}\left(x+\frac{1}{x}\right)+C$

   c) $\tan^{-1}\left(3x^2+2x\right)+C$ d) $\tan^{-1}\left(x^2+1\right)+C$

2. The projections of a vector on the three coordinate axes are 6, -3 and 2, respectively. The direction cosines of the vector are **[1]**

   a) $-\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$ b) $\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$

   c) none of these d) 6, -3, 2

3. The area bounded by the parabola x = 4 - y^2 and y-axis, in square units, is **[1]**

   a) $\frac{33}{2}$ b) $\frac{3}{32}$

   c) $\frac{32}{3}$ d) $\frac{16}{3}$

4. The magnitude of the vector $6\hat{i}+2\hat{j}+3\hat{k}$ is **[1]**

   a) 7 b) 5

   c) 12 d) 1

5. From each of the four married couples, one of the partners is selected at random. The probability that those selected are of the same sex is **[1]**

a) $\frac{1}{8}$

b) $\frac{1}{16}$

c) $\frac{1}{2}$

d) $\frac{1}{4}$

6. The area bounded by the curve y = f (x), x-axis, and the ordinates x = I and x - b is (b - 1) sin (3b + 4). Then, f (x) is **[1]**

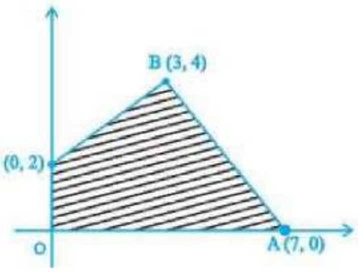
a) sin (3x + 4)

b) sin (3x + 4) + 3 (x -1) cos (3x + 4)

c) None of these

d) (x -1) cos(3x + 4)

7. Feasible region (shaded) for a LPP is shown in Figure. Maximize Z = 5x + 7y. **[1]**



a) 45

b) 49

c) 47

d) 43

8. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accidents are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver? **[1]**

a) $\frac{1}{52}$

b) $\frac{3}{52}$

c) $\frac{5}{52}$

d) $\frac{1}{4}$

9. Variable separation method can be used to solve First Order, First Degree Differential Equations in which y' is of the form. **[1]**

a) $y^2 = cos\ (g\,(y))$

b) $y^2 = sin\ (h\,(x))$

c) y' = h(x)g(y)

d) $y^3 = g\,(y)$

10. For any vector $\vec{a}$, the value of $(\vec{a} \times \hat{i})^2 + (\vec{a} \times \hat{j})^2 + (\vec{a} \times \hat{k})^2$ is equal to **[1]**

a) $\overrightarrow{3a^2}$

b) $\overrightarrow{4a^2}$

c) $\overrightarrow{a^2}$

d) $\overrightarrow{2a^2}$

11. $\int \frac{(x+1)(x+\log x)^2}{x} dx = ?$ **[1]**

a) $\frac{x^3}{3}+\frac{x^2}{2}+x+C$

b) $\frac{x^2}{2}+x+C$

c) None of these

d) $\frac{1}{3}(x+\log x)^3+C$

12. Consider the following statements [1]

i. The general solution of $\frac{dy}{dx}$ = f(x) + x is of the dx form y = g(x) + C, where C is an arbitrary constant.

ii. The degree of $\left(\frac{dy}{dx}\right)^2$ = f(x) is 2.

Which of the above statement(s) is/are correct?

a) Only (ii)

b) Only (i)

c) Both (i) and (ii)

d) Neither (i) nor (ii)

13. The solution of the differential equation $x\frac{dy}{dx}+2y=x^2$ is [1]

a) $y=\frac{x^4+c}{x^2}$

b) $y=\frac{x^4+c}{4x^2}$

c) $y=\frac{x^2+c}{4x^2}$

d) $y=\frac{x^2}{4}+c$

14. If A is an invertible matrix of order 2, then det (A^{-1}) is equal to [1]

a) 0

b) $\frac{1}{\det(A)}$

c) det (A)

d) 1

15. If xy - $\log_e$ y = 1 satisfies the equation $x\left(yy_2+y_1^2\right)-y_2+\lambda yy_1=0$, then λ = [1]

a) 1

b) 3

c) -3

d) none of these

16. The value of $\cos^{-1}(-1) - \sin^{-1}(1)$ is [1]

a) $\frac{3\pi}{2}$

b) π

c) $-\frac{3\pi}{2}$

d) $\frac{\pi}{2}$

17. The lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-1}{3}=\frac{y-2}{4}=\frac{z-3}{5}$ are [1]

a) none of these

b) parallel

c) intersecting

d) skew

18. y = 2 cos x + 3 sin x satisfies which of the following differential equations? [1]

i. $\frac{d^2y}{dx^2}$ + y = 0

ii. $\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx}$ = 0

Select the correct answer using the codes given below.

a) Only (ii)
b) Neither (i) nor (ii)
c) Only (i)
d) Both (i) and (ii)

19. **Assertion (A):** The maximum value of Z = 11x + 7y subject to the constraints 2x + y $\leq$ 6; x $\leq$ 2; x $\geq$ 0, y $\geq$ 0 occurs at the comer point (0, 6). **[1]**
Reason (R): If the feasible region of the given LPP is bounded, then the maximum and minimum value of the objective function occurs at corner points.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** |sin x| is a continuous function. **[1]**
Reason (R): If f(x) and g(x) both are continuous functions, then gof(x) is also a continuous funtion.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Differentiate the function with respect to x: sin(x^x) **[2]**

22. Find the general solution for differential equation: $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ **[2]**

23. Find the direction cosines of the lines, connected by the relations: l + m + n = 0 and 2lm + 2ln - mn = 0. **[2]**

OR

The equations of a line are given by $\frac{4-x}{3} = \frac{y+3}{3} = \frac{z+2}{6}$. Write the direction cosines of a line parallel to this line.

24. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ **[2]**

25. A and B are two events such that P(A) = $\frac{1}{2}$ P(B) = $\frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$. [2]
Find P(A'|B')

Section C

26. Solve the Linear Programming Problem graphically: [3]
Minimize Z = 30x + 20y Subject to
$x + y \leq 8$
$x + 4y \geq 12$
$5x + 8y = 20$
$x, y \geq 0$

27. Evaluate the integral: $\int(3x+1)\sqrt{4-3x-2x^2}dx$ [3]

OR

Evaluate the integral: $\int \frac{1}{2+\cos x}dx$

28. Find the area of a minor segment of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{2}$. [3]

OR

Find the area bounded by the parabola $y = 2 - x^2$ and the straight line y + x = 0.

29. Find the area of the region bounded by $x^2 = 16y$, y = l, y = 4 and the y-axis in the first quadrant. [3]

30. Differentiating the function w.r.t. x: $\tan^{-1}\left(\frac{e^{2x}+1}{e^{2x}-1}\right)$. [3]

31. Find the direction cosines of the sides of the triangle whose vertices are (3, 5, -4), (-1, 1, 2) and (-5, -5, -2). [3]

OR

By computing the shortest distance determine whether the pairs of lines intersect or not:
$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$

Section D

32. For the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find numbers a and b such that $A^2 + aA + bI = 0$. [5]

OR

The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third number, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

33. If $\vec{a}, \vec{b}, \vec{c}$ are unit vector such that $\vec{a} + \vec{b} + \vec{c} = 0$ find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ **[5]**

34. Given, A = {2, 3, 4}, B = {2, 5, 6, 7}. Construct an example of each of the following: **[5]**

a. an injective mapping from A to B

b. a mapping from A to B which is not injective

c. a mapping from B to A.

OR

Let A and B be two sets. Show that f: $A \times B \to B \times A$ such that f(a, b) = (b, a) is

(i) injective

(ii) bijective

35. Integrate the function $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx$, x $\in$ [0,1] **[5]**

Section E

36. **Read the text carefully and answer the questions:** **[4]**

Naina is creative she wants to prepare a sweet box for Diwali at home. She took a square piece of cardboard of side 18 cm which is to be made into an open box, by cutting a square from each corner and folding up the flaps to form the box. She wants to cover the top of the box with some decorative paper. Naina is interested in maximizing the volume of the box.

(i) Find the volume of the open box formed by folding up the cutting each corner with x cm.

(ii) Naina is interested in maximizing the volume of the box. So, what should be the side of the square to be cut off so that the volume of the box is maximum?

(iii) Verify that volume of the box is maximum at x = 3 cm by second derivative test?

OR

Find the maximum volume of the box.

37. **Read the text carefully and answer the questions:** **[4]**

The nut and bolt manufacturing business has gained popularity due to the rapid Industrialization and introduction of the Capital-Intensive Techniques in the Industries that are used as the Industrial fasteners to connect various machines and structures. Mr. Suresh is in Manufacturing business of Nuts and bolts. He produces three types of bolts, x, y, and z which he sells in two markets. Annual sales (in ₹) indicated below:

Markets	**Products**		
	x	y	z
I	10000	2000	18000
II	6000	20000	8000

(i) If unit sales prices of x, y and z are ₹2.50, ₹1.50 and ₹1.00 respectively, then find the total revenue collected from Market-I &II.

(ii) If the unit costs of the above three commodities are ₹2.00, ₹1.00 and 50 paise respectively, then find the cost price in Market I and Market II.

(iii) If the unit costs of the above three commodities are ₹2.00, ₹1.00 and 50 paise respectively, then find gross profit from both the markets.

OR

If matrix A = $[a_{ij}]_{2\times2}$ where a_{ij} = 1, if i $\neq$ j and a_{ij} = 0, if i = j then find A^2.

38. **Read the text carefully and answer the questions:** **[4]**

Mr. Ajay is taking up subjects of mathematics, physics, and chemistry in the examination. His probabilities of getting a grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(i) Find the probability that Ajay gets Grade A in all subjects.

(ii) Find the probability that he gets Grade A in no subjects.

SOLUTION

Section A

1. **(b)** $\tan^{-1}\left(x + \frac{1}{x}\right) + C$

Explanation: Divide num. and deno. by x^2

Substitute $x + \frac{1}{x} = t$ then $(1 - \frac{1}{x^2})dx = dt$

$\Rightarrow \int \frac{dt}{t^2+1}$

$\Rightarrow \tan^{-1}(x + \frac{1}{x}) + C$

2. **(b)** $\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$

Explanation: Projection of vector on coordinate axes are
$x_2 - x_1, y_2 - y_1, z_2 - z_1$
$\Rightarrow x_2 - x_1 = 6, y_2 - y_1 = -3, z_2 - z_1 = 2$

Now, $\sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2} = \sqrt{36 + 9 + 4} = 7$

So, the direction cosines of the vector are $\frac{6}{7}, -\frac{3}{7}, \frac{2}{7}$

3. **(c)** $\frac{32}{3}$

Explanation: The area bounded by parabola $x = 4 - y^2$ and y-axis
As parabola bounded by y-axis $x = 0$
$\Rightarrow 4 - y^2 = 0$
$\Rightarrow y = \pm 2$

$\int_{-2}^{2}\left(4 - y^2\right)dy$

$\left[4y - \frac{y^3}{3}\right]_{2}^{2}$

$16 - \left(\frac{8}{3} + \frac{8}{3}\right)$

$= \frac{32}{3}$

4. **(a)** 7

Explanation: 7

5. **(a)** $\frac{1}{8}$

Explanation: Here, s = {(M M M M), (F F F F), ...}

Clearly, n (s) = 16

$\therefore$ Required proability = P [(M M M M) or (F F F F)]

= P [(M M M M) + (F F F F)]

$= \frac{2}{16} + \frac{2}{16} = \frac{4}{16} = \frac{1}{8}$

6. **(b)** sin (3x + 4) + 3 (x -1) cos (3x + 4)

Explanation: Given that area bounded by the curve x-axis,

x = 1 and x = b

$\Rightarrow \int_1^b y dx = \int_1^b f(x) dx$

$\Rightarrow \int_1^b y dx - [A]_1^b$

$\Rightarrow \int_1^b f(x) dx = (b-1)\sin(3b+4)$

$\Rightarrow f(x) - \frac{d}{dx}[(x-1)\sin(3x+4)]$

$\Rightarrow$ 3(x - 1) cos (3x + 4) + sin (3x + 4)

7. **(d)** 43

Explanation:

Corner points	Z = 5x + 7y
O(0,0)	0
B (3,4)	43
A(7,0)	35
C(0,2)	14

Hence the maximum value is 43

8. **(a)** $\frac{1}{52}$

Explanation: Let, E_1, E_2 and E_3 are events of selection of a scooter driver, car driver and truck driver respectively.

$\therefore P(E_1) = \frac{2000}{12000} = \frac{1}{6}, P(E_2) = \frac{4000}{12000} = \frac{1}{3}, P(E_3) = \frac{6000}{12000} = \frac{1}{2}.$

Let A = event that the insured person meet with the accident.

$\therefore$ P(A|E_1) = 0.01, P(A|E_2) = 0.03, P(A|E_3) = 0.15

$$\Rightarrow P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)P(A|E_2)+P(E_3)P(A|E_3)}$$

$$= \frac{\frac{1}{6}\times 0.01}{\frac{1}{6}\times 0.01+\frac{1}{3}\times 0.03+\frac{1}{2}\times 0.15} = \frac{1}{52}$$

9. **(c)** y' = h(x)g(y)

Explanation: y' = h(x)g(y) since we can segregate functions of y with dy and x with dx.

$\frac{dy}{dx} = h(x)g(y)$ and $\frac{dy}{g(y)} = h(x)dx$

10. **(d)** $2\vec{a}^2$

Explanation: Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, then

$\vec{a}\times\hat{i} = \left(a_1\hat{i} + a_2\hat{j} + a_3\hat{k}\right)\times\hat{i} = a_1\hat{i}\times\hat{i} + a_2\hat{j}\times\hat{i} + a_3\hat{k}\times\hat{i}$

$\Rightarrow \vec{a}\times\hat{i} = 0 - a_2\hat{k} + a_3\hat{j}(\because \hat{i}\times\hat{i} = 0, \hat{j}\times\hat{i} = -\hat{k}, \hat{k}\times\hat{i} = \hat{j})$

$\Rightarrow \vec{a}\times\hat{i} = -a_2\hat{k} + a_3\hat{j}$

$\Rightarrow |\vec{a}\times\hat{i}|^2 = a_2^2 + a_3^2$

Similarly, we get

$\Rightarrow |\vec{a}\times\hat{j}|^2 = a_1^2 + a_3^2$

$\Rightarrow |\vec{a}\times\hat{k}|^2 = a_1^2 + a_2^2$

$\therefore |\vec{a}\times\hat{i}|^2 + |\vec{a}\times\hat{j}|^2 + |\vec{a}\times\hat{k}|^2 = a_2^2 + a_3^2 + a_1^2 + a_3^2 + a_1^2 + a_2^2$

$= 2\left(a_1^2 + a_2^2 + a_3^2\right) = 2|\vec{a}|^2(\because |\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}).$

11. **(d)** $\frac{1}{3}(x + \log x)^3 + C$

Explanation: Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c; \int \frac{1}{1+x^2}dx = \tan^{-1}x + c$

Therefore,

Put $x^1 + \log x = t\left(1 + \frac{1}{x}\right)dx = dt \Rightarrow \left(\frac{x+1}{x}\right)dx = dt$

$= \int t^2 dt$

$= \frac{t^2}{3} + c$

$= \frac{(x+\log x)^3}{3} + c$

12. **(c)** Both (i) and (ii)

Explanation:

i. We have, $\frac{dy}{dx} = f(x) + x \Rightarrow dy = [f(x) + x]dx$

On integrating both sides, we get

$\int dy = \int [f(x) + x]dx \Rightarrow y = \int f(x)\,dx + \frac{x^2}{2} + C$

Let $g(x) = \int f(x)dx + \frac{x^2}{2}$

Thus, general solution is of the form $y = g(x) + C$

ii. Consider the given differential equation $\left(\frac{dy}{dx}\right)^2 = f(x)$

Clearly, the highest order derivative occurring in the differential equation is $\frac{dy}{dx}$ and its highest power is 2.

iii. Also, given equation is polynomial in the derivative. So the degree of a differential equation is 2.

13. **(b)** $y = \frac{x^4 + c}{4x^2}$

Explanation: Here,

Integrating factor, I.F $= e^{\int \frac{2}{x}dx} = e^{2\log x} = e^{\log x^2} = x^2$

Therefore, the solution is $y.x^2 = \int x^2 \cdot x dx = \frac{x^4}{4} + k$,

i.e. $y = \frac{x^4 + c}{4x^2}$

14. **(b)** $\frac{1}{\det(A)}$

Explanation: We know that, $A^{-1} = \frac{1}{|A|}\text{Adj}(A)$

So, $\left|A^{-1}\right| = \left|\frac{1}{|A|}\text{Adj}(A)\right|$

$$= \frac{1}{|A|^n} |\mathrm{Adj}(A)|$$

$$= \frac{1}{|A|^n} |A|^{n-1} = \frac{1}{|A|^1}$$

$$= \frac{1}{|A|^1}$$

{since adj(A) is of order n and $|\mathrm{Adj}(A)| = |A|^{n-1}$}

15. **(d)** none of these

Explanation: $xy - \log_e y = 1$

Differentiating both sides we get

$$xy_1 + y - \frac{y_1}{y} = 0$$

$\Rightarrow xyy_1 + y^2 - y_1 = 0$

Again differentiating both sides we get

$$x\left(yy_2 + y_1^2\right) + 2yy_1 - y_2 = 0$$

On comparing with $x\left(yy_2 + y_1^2\right) + \lambda yy_1 - y_2$ we get $\lambda = 2$

16. **(d)** $\frac{\pi}{2}$

Explanation: Let $\cos^{-1}(-1) = A \Rightarrow \cos A = -1 \Rightarrow \cos A = \cos\pi \therefore A = \pi$

and $\sin^{-1}(1) = B \Rightarrow \sin B = 1 \Rightarrow \sin B = \sin\left(\frac{\pi}{2}\right)$

$\therefore B = \left(\frac{\pi}{2}\right)$

$\therefore \cos^{-1}(-1) - \sin^{-1}(1) = \pi - \frac{\pi}{2} = \frac{\pi}{2}$.Which is the required solution.

17. **(c)** intersecting

Explanation: Here $(a_1, b_1, c_1) = (2, 3, 4)$

and, $(a_2, b_2, c_2) = (3, 4, 5)$

Consider $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

We get

$$\begin{vmatrix} 0 & 0 & 0 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 0$$

Since the shortest distance is zero hence the lines are intersecting.

18. **(c)** Only (i)

Explanation: Given differential equation is

$y = 2 \cos x + 3 \sin x$ ...(i)

Now, $\frac{dy}{dx}$ = = $-2\sin x + 3\cos x$

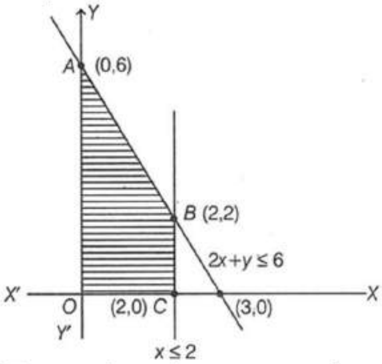
$= -(2 \cos x + 3 \sin x) = -y$ [from Eq. (i)]

$\therefore \frac{d^2y}{dx^2} + y = 0$

So, only Statement (i) is correct.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Assertion: The corresponding graph of the given LPP is



From the above graph, we see that the shaded region is the feasible region OABC which is bounded.

$\therefore$ The maximum value of the objective function Z occurs at the corner points. The corner points are O(0, 0), A(0, 6), B(2, 2), C(2, 0).

The values of Z at these comer points are given by

Comer point	Corresponding value of $Z = 11x + 7y$
(0, 0)	0
(0, 6)	42 ← Maximum
(2, 2)	36
(2, 0)	22

Thus, the maximum value of Z is 42 which occurs at the point (0, 6).

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Section B

21. Let y = sin x^x

$\Rightarrow$ $\sin^{-1} y = x^x$ ...(i)

Taking log on both sides,

$\log(\sin^{-1} y) = \log x^x$

$\Rightarrow$ $\log(\sin^{-1} y) = x \log x$

Differentiating both sides with respect to x,

$$\Rightarrow \frac{1}{\sin^{-1} y}\frac{dy}{dx}\left(\sin^{-1}y\right) = x\frac{d}{dx}\log x + \log x\frac{d}{dx}x$$

$$\Rightarrow \frac{1}{\sin^{-1} y} \times \left(\frac{1}{\sqrt{1-y^2}}\right)\frac{dy}{dx} = x\left(\frac{1}{x}\right) + \log x$$

$$\Rightarrow \frac{dy}{dx} = \sin^{-1} y\sqrt{1-y^2}(1+\log x)$$

$$\Rightarrow \frac{dy}{dx} = \sin^{-1}\left(\sin x^x\right)\sqrt{1-\left(\sin x^x\right)^2}(1+\log x)$$

$\therefore \frac{dy}{dx} = x^x \cos x^x(1 + \log x)$ [using equation (i)]

22. We have, $\frac{dy}{dx} = e^x e^{-y} + x^2 e^{-y}$

$$\Rightarrow \frac{dy}{dx} = e^{-y}\left(e^x + x^2\right)$$

$$\Rightarrow \frac{dy}{e^{-y}} = \left(e^x + x^2\right)dx$$

Integrating both the sides we get:

$$\Rightarrow \int\frac{dy}{e^{-y}} = \int\left(e^x + x^2\right)dx + c$$

$$\Rightarrow e^y = e^x + \frac{x^3}{3} + c$$

$$e^y = e^x + \frac{x^2}{3} + c$$

23. We are given that: l + m + n = 0...(i)

2lm + 2ln − nm = 0…(ii)

From (i), we get

l = -m - n

Substituting l = -m - n in (ii), we get

2(-m -n) m + 2(-m - n) n - m n = 0

$\Rightarrow$ $-2m^2 - 2mn - 2mn - 2n^2 - mn = 0$

$\Rightarrow$ $2m^2 + 2n^2 + 5mn = 0$

$\Rightarrow$ (m + 2n) (2m + n) = 0

$\Rightarrow$ m = -2n, $-\frac{n}{2}$

If m = -2 n, then from (i), we get l = n

If m = $-\frac{n}{2}$, then from (i), we get l = $-\frac{n}{2}$.

Thus, the direction ratios of the two lines are proportional to n, -2n, n and $-\frac{n}{2}$, $-\frac{n}{2}$, n.

Hence the required direction cosines are

$\pm\frac{1}{\sqrt{6}}, \pm\frac{-2}{\sqrt{6}}, \pm\frac{1}{\sqrt{6}}$

$\pm\frac{-1}{\sqrt{6}}, \pm\frac{-1}{\sqrt{6}}, \pm\frac{2}{\sqrt{6}}$

OR

We have

$\frac{4-x}{3} = \frac{y+3}{3} = \frac{z+2}{6}$

The equation of the given line can be re-written as

$\frac{x-4}{-3} = \frac{y+3}{3} = \frac{z+2}{6}$

The direction ratios of the line parallel to the given line are proportional to -3, 3, 6.

Hence, the direction cosines of the line parallel to the given line are proportional to

$\frac{-3}{\sqrt{(-3)^2+3^2+6^2}}, \frac{3}{\sqrt{(-3)^2+3^2+6^2}}, \frac{6}{\sqrt{(-3)^2+3^2+6^2}}$

$= \frac{-1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}$

24. Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = y$

$\Rightarrow \cos y = \frac{\sqrt{3}}{2}$

$\Rightarrow \cos y = \cos\frac{\pi}{6}$

$\Rightarrow y = \frac{\pi}{6}$

Since, the principal value branch of $\cos^{-1}$ is $[0, \pi]$.

Therefore, principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$.

25. $P\left(A'|B'\right) = \frac{P(A' \cap B')}{P\left(B'\right)} = \frac{1-P(AUB)}{1-P(B)}$

$$= \frac{1-[P(A)+P(B)-P(A \cap B)]}{1-P(B)}$$

$$= \frac{1-\left[\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\right]}{1-\frac{1}{3}}$$

$$= \frac{1-\left[\frac{5}{6}-\frac{1}{4}\right]}{\frac{2}{3}}$$

$$= \frac{1-\frac{14}{24}}{\frac{2}{3}}$$

$$= \frac{\frac{10}{24}}{\frac{2}{3}}$$

$$= \frac{30}{48}$$

$$= \frac{5}{8}$$

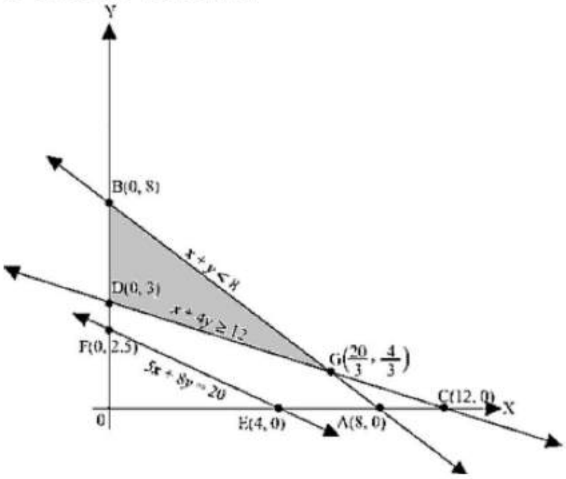
Section C

26. First, we will convert the given inequations into equations, we obtain the following equations:
$x + y = 8$, $x + 4y = 12$, $x = 0$ and $y = 0$
$5x + 8y = 20$ is already an equation.
Region represented by $x + y \le 8$ The line $x + y = 8$ meets the coordinate axes at A(8,0) and B(0,8) respectively. By joining these points we obtain the line $x + y = 8$. Clearly (0,0) satisfies the inequation $x + y \le 8.50$, the region in x y plane which contain the origin represents the solution set of the

inequation x + y $\leq$ 8.
Region represented by x + 4 y $\geq$ 12 :
The line x + 4 y = 12 meets the coordinate axes at C(12,0) and D(0,3) respectively. By joining these points we obtain the line x + 4 y = 12 . Clearly (0,0) satisfies the inequation x + 4 y $\geq$ 12 . So, the region in x y plane which does not contain the origin represents the solution set of the inequation x + 4 y $\geq$ 12.

The line 5 x + 8 y = 20 is the line that passes through E(4,0) and $F\left(0, \frac{5}{2}\right)$

Region represented by x $\geq$ 0 and y $\geq$ 0 :
since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0 and y $\geq$ 0.
The feasible region determined by subject to the constraints arex + y $\leq$ 8, x + 4 y $\geq$ 12,5 x + 8 y = 20 and the non-negative restrictions , x $\geq$ 0 and y $\geq$ 0 are as follows.



The corner points of the feasible region are B(0,8), D(0,3), $G\left(\frac{20}{3}, \frac{4}{3}\right)$

The values of objective function at corner points are as follows:
Corner point: Z = 30x +20y
B(0,8): 160
D(0,3): 60

$G\left(\frac{20}{3}, \frac{4}{3}\right)$: 266.66

Therefore, the minimum value of objective function Z is 60 at the point D(0,3) . Hence, x = 0 and y = 3 is the optimal solution of the given LPP.
Thus, the optimal value of objective function Z is 60.

27. Let the given integral be,

$$I = \int (3x + 1)\sqrt{4 - 3x - 2x^2}dx$$

$$\text{Let } (3x + 1) = A\frac{d}{dx}\left(4 - 3x - 2x^2\right) + B$$

$\Rightarrow (3x + 1) = A(-3 - 4x) + B$

$\Rightarrow (3x + 1) = -4Ax + (B - 3A)$

$\Rightarrow 3 = -4A$ and $(B - 3A) = 1$

$\Rightarrow A = -\frac{3}{4}$ and $B = -\frac{5}{4}$

$$\Rightarrow I = -\frac{3}{4}\int(-3-4x)\sqrt{4-3x-2x^2}dx - \frac{5}{4}\int\sqrt{4-3x-2x^2}dx$$

Let $I = -\frac{3}{4}I_1 - \frac{5}{4}I_2$ .....(i)

Now,

$$I_1 = \int(-3-4x)\sqrt{4-3x-2x^2}dx$$

Let $(4 - 3x - 2x^2) = t$ or $(-3 - 4x)dx = dt$

$$\Rightarrow I_1 = \int\sqrt{t}dt$$

$$= \frac{2}{3}t^{\frac{3}{2}} + c_1$$

$$\Rightarrow I_1 = \frac{2}{3}\left(4-3x-2x^2\right)^{\frac{3}{2}} + c_1$$

Now, $I_2 = \int\sqrt{4-3x-2x^2}dx$

$$= \int\sqrt{2\left(2-\frac{3}{2}x-x^2\right)}dx$$

$$= \sqrt{2}\int\sqrt{\left(\frac{17}{4}-\frac{9}{4}-\frac{3}{2}x-x^2\right)}dx$$

$$= \sqrt{2}\int\sqrt{\left[\left(\frac{\sqrt{17}}{2}\right)^2-\left(\frac{9}{4}+\frac{3}{2}x+x^2\right)\right]}dx$$

$$= \sqrt{2}\int\sqrt{\left[\left(\frac{\sqrt{17}}{2}\right)^2-\left(x+\frac{3}{2}\right)^2\right]}dx$$

$$= \sqrt{2}\sin\left(\frac{x+\frac{3}{2}}{\frac{\sqrt{17}}{2}}\right) + c_2$$

$$= \sqrt{2}\sin\left(\frac{2x+3}{\sqrt{17}}\right) + c_2$$

Using (i), we get

$$I = -\frac{3}{4} \times \frac{2}{3}\left(4 - 3x - 2x^2\right)^{\frac{3}{2}} - \frac{5}{4} \times \sqrt{2}\sin\left(\frac{2x+3}{\sqrt{17}}\right) + C$$

$$\therefore\ I = -\frac{1}{2}\left(4 - 3x - 2x^2\right)^{\frac{3}{2}} - \frac{5\sqrt{2}}{4}\sin\left(\frac{2x+3}{\sqrt{17}}\right) + C$$

OR

Let the given integral be,

$$I = \int \frac{1}{2+\cos x}dx$$

Putting $\cos x = \dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$$\therefore I = \int \frac{1}{2+\left(\dfrac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}\right)}dx$$

$$= \int \frac{\left(1+\tan^2\frac{x}{2}\right)}{2\left(1+\tan^2\frac{x}{2}\right)+1-\tan^2\left(\frac{x}{2}\right)}dx$$

$$= \int \frac{\sec^2\left(\frac{x}{2}\right)}{2+2\tan^2\left(\frac{x}{2}\right)+1-\tan^2\left(\frac{x}{2}\right)}dx$$

$$= \int \frac{\sec^2\left(\frac{x}{2}\right)}{2 + 2\tan^2\left(\frac{x}{2}\right) + 1 - \tan^2\left(\frac{x}{2}\right)} dx$$

$$= \frac{\sec^2\left(\frac{x}{2}\right)}{3 + \tan^2\left(\frac{x}{2}\right)} dx$$

Putting $\tan\frac{x}{2} = t$

$$\Rightarrow \frac{1}{2}\sec^2\left(\frac{x}{2}\right) dx = dt$$

$$\Rightarrow \sec^2\left(\frac{x}{2}\right) dx = 2dt$$

$$\therefore I = \int \frac{2}{3 + t^2} dt$$

$$= 2\int \frac{1}{t^2 + (\sqrt{3})^2} dt$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{t}{\sqrt{3}}\right) + C$$

$$= \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{\tan\frac{x}{2}}{\sqrt{3}}\right) + C \left[\because t = \tan\frac{x}{2}\right]$$

28. Solving the equation $x^2 + y^2 = a^2$ and $x = \frac{a}{2}$, we obtain their points of intersection which are $\left(\frac{a}{2}, \sqrt{3}\frac{a}{2}\right)$ and $\left(\frac{a}{2} - \frac{\sqrt{3}a}{2}\right)$.

Hence, we get

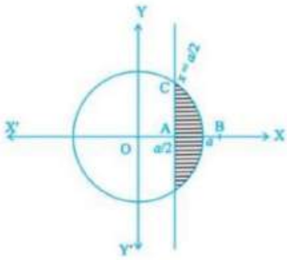
Required Area

$$= 2\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{2}\right]_{\frac{a}{2}}^{a}$$

$$= 2\left[\frac{a^2}{2}\cdot\frac{\pi}{2} - \frac{a}{4}\cdot a\frac{\sqrt{3}}{2} - \frac{a^2}{2}\cdot\frac{\pi}{6}\right]$$

$$= \frac{a^2}{12}\left(6\pi - 3\sqrt{3} - 2\pi\right)$$

$$= \frac{a^2}{12}\left(4\pi - 3\sqrt{3}\right) \text{ sq units}$$



OR

To find area bounded by

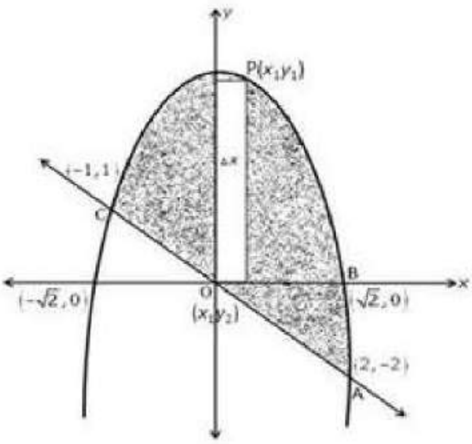
$y = 2 - x^2$ ...(i)

and $y + x = 0$

Equation (i) represents a parabola with vertex (0, 2) and downward, meets axes at $(\pm\sqrt{2}, 0)$

Equation (ii) represents a line passing through (0, 0) and (2, -2). The points of intersection of line and parabola are (2, -2) and (-1, 1)

A rough sketch of curves is as below:-



Shaded region is sliced into rectangles with area = $(y_1 - y_2)\triangle x$. It slides from $x = -1$ to $x = 2$, so

Requried area of the shaded region =Area of Region ABPCOA

$$A = \int_{-1}^{2}\left(y_1 - y_2\right)dx$$

$$= \int_{-1}^{2} \left(2 - x^2 + x\right) dx$$

$$= \left[2x - \frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{2}$$

$$= \left[\left(4 - \frac{8}{3} + 2\right) - \left(-2 + \frac{1}{3} + \frac{1}{2}\right)\right]$$

$$= \left[\frac{10}{3} + \frac{7}{6}\right]$$

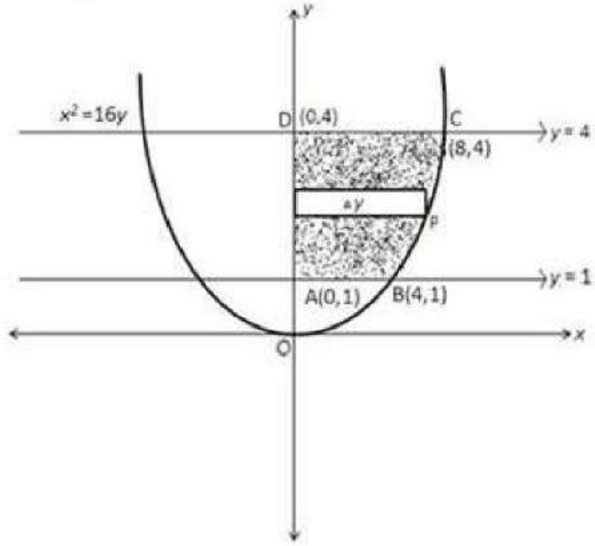
$$= \frac{27}{6}$$

$A = \frac{9}{2}$ sq. units

29. To find region in first quadrant bounded by y = 1, y = 4 and y-axis and $x^2 = 16y$.........(i)

Equation (i) represents a parabola with vertex (0, 0) and axes as y=axis.

A rough sketch of the curves is as under:-



Shaded region is required area it is sliced in rectangles of area $x\triangle y$ which slides from y = 1 to y = 4, so

Required area = Area of the Region ABCDA which is given as

$$A = \int_1^4 x dy$$

$$= \int_1^4 4\sqrt{y} dy$$

$$= 4 \cdot \left[\frac{2}{3} y\sqrt{y}\right]_1^4$$

$$= 4 \cdot \left[\left(\frac{2}{3} \cdot 4\sqrt{4}\right) - \left(\frac{2}{3} \cdot 1 \cdot \sqrt{1}\right)\right]$$

$$= 4\left[\frac{16}{3} - \frac{2}{3}\right]$$

$$A = \frac{56}{3} \text{ sq. units}$$

30. The given function is: $\tan^{-1}\left\{\frac{e^{2x}+1}{e^{2x}-1}\right\}$

$$\text{or } \tan^{-1}\left\{\frac{1+e^{2x}}{-\left(1-e^{2x}\right)}\right\}$$

$$\text{or } -\tan^{-1}\left\{\frac{1+e^{2x}}{1-e^{2x}}\right\}$$

Putting $e^{2x} = \tan\theta$

$\theta = \tan^{-1}(e^{2x})$ ...(i)

Putting $e^{2x} = \tan\theta$ in the equation, we have

$$\Rightarrow -\tan^{-1}\left\{\frac{1+\tan\theta}{1-\tan\theta}\right\}$$

$$= -\tan^{-1}\left\{\frac{\tan\frac{\pi}{4}+\tan\theta}{1-\tan\frac{\pi}{4}\tan\theta}\right\}$$

$$= -\tan^{-1}\left\{\tan\left(\frac{\pi}{4}+\theta\right)\right\}$$

$$= -\left(\frac{\pi}{4}+\theta\right)$$

$$= -\frac{\pi}{4} - \theta$$

$$= -\frac{\pi}{4} - \tan^{-1}\left(e^{2x}\right)$$

Now we can see that $\tan^{-1}\left\{\frac{e^{2x}+1}{e^{2x}-1}\right\} = -\frac{\pi}{4} - \tan^{-1}\left(e^{2x}\right)$

Now, Differentiating

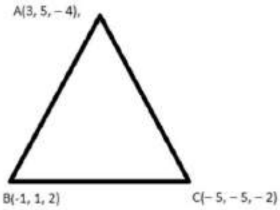
$$\Rightarrow \frac{d}{dx}\left(-\frac{\pi}{4} - \tan^{-1}\left(e^{2x}\right)\right)$$

$$= \frac{d}{dx}\left(-\frac{\pi}{4}\right) - \frac{d}{dx}\left(\tan^{-1}\left(e^{2x}\right)\right)$$

$$= 0 - \frac{d}{dx}\left(\tan^{-1}\left(e^{2x}\right)\right)\frac{d}{dx}e^{2x}\frac{d}{dx}(2x)$$

$$= -\left(\frac{1}{1+\left(e^{2x}\right)^2}\right)e^{2x}2$$

$$= \frac{-2e^{2x}}{1+e^{4x}}$$

31. 

The direction cosines of the two points passing through $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by

$(x_2 - x_1), (y_2 - y_1), (z_2 - z_1)$

And the direction cosines of the line AB is

$$\frac{\left(x_2-x_1\right)}{AB}, \frac{\left(y_2-y_1\right)}{AB}, \frac{\left(z_2-z_1\right)}{AB}$$

Where $AB = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2}$

Here A = (3, 5, -4)	B = (-1, 1, 2)	C = (-5, -5, -2)
B = (-1, 1, 2)	C = (-5, -5, -2)	A = (3, 5, -4)
Direction ratios of AB	Direction ratio of BC	Direction ratios of CA
(-1-3), (1 - 5), (2 - (-4))	(-5 + 1), (-5 - 1), (-2, -2)	(3 + 5), (5 + 5), (-4 + 2)
= -4, -4, 6	= (-4, -6, -4)	= (8, 10, -2)
$\sqrt{AB} = \sqrt{(68)} = 2\sqrt{17}$	$\sqrt{BC} = \sqrt{(68)} = 2\sqrt{17}$	$\sqrt{CA} = \sqrt{(168)} = 2\sqrt{42}$

Direction Cosines of AB are	Direction Cosines of BC are	Direction Cosines of CA are
$\frac{-4}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}, \frac{6}{2\sqrt{17}}$	$\frac{-4}{2\sqrt{17}}, \frac{-6}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}$	$\frac{8}{2\sqrt{42}}, \frac{10}{2\sqrt{42}}, \frac{-2}{2\sqrt{42}}$
$\frac{-2}{\sqrt{17}}, \frac{-2}{\sqrt{17}}, \frac{3}{\sqrt{17}}$	$\frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$	$\frac{4}{\sqrt{42}}, \frac{5}{\sqrt{42}}, \frac{-1}{\sqrt{42}}$

OR

Given that,

$\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$

Comparing the given equations with the equations

$\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$

We get,

$\vec{a_1} = \hat{i} + \hat{j} - \hat{k}$

$\vec{a_2} = 4\hat{i} - \hat{k}$

$\vec{b_1} = 3\hat{i} - \hat{j}$

$\vec{b_2} = 2\hat{i} + 3\hat{k}$

$\therefore \vec{a_2} - \vec{a_1} = 3\hat{i} - \hat{j}$

and $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ 2 & 0 & 3 \end{vmatrix}$

$= -3\hat{i} - 9\hat{j} + 2\hat{k}$

$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = (3\hat{i} - \hat{j}) \cdot (-3\hat{i} - 9\hat{j} + 2\hat{k})$

$= -9 + 9$

$= 0$

We observe

$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = 0$

Thus, the given lines intersect.

Section D

32. Given: $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$$\therefore A^2 = A.A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$$

$\therefore A^2 + aA + bI_2 = 0$

$$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 11+3a+b & 8+2a+0 \\ 4+a+0 & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore$ We have $11 + 3a + b = 0$....(i)

$8 + 2a + 0 = 0$ ...(ii)

$\Rightarrow 2a = -8$

$\Rightarrow a = -4$

Here a = -4 satisfies 4 + a + 0 = 0 also, therefore a = -4

Putting a = -4 in eq. (i), 11 - 12 + b = 0 $\Rightarrow b - 1 = 0 \Rightarrow b = 1$

Here also b = 1 satisfies 3 + a + b = 0, therefore b = 1

Therefore, a = - 4 and b = 1

OR

Let first ,second and third number be denoted by x,y and z, respectively.Then, according to given conditions,we have,

x + y + z = 6

y + 3z = 11

x + z = 2y or x-2y+z=0

This system can be written as AX = B whose

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$|A| = 1(1+6) - (0-3) + (0-1) = 9 \neq 0$

$A_{11} = 7, A_{12} = 3, A_{13} = -1$

$A_{21} = -3, A_{22} = 0, A_{23} = 3$

$A_{31} = 2, A_{32} = -3, A_{33} = 1$

$$adjA = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} adjA = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 42 - 33 + 0 \\ 18 + 0 + 0 \\ -6 + 33 + 0 \end{bmatrix} = = \frac{1}{9}\begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, x=1, y=2, z=3

33. $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 1, \left|\vec{c}\right| = 1$

$\vec{a} + \vec{b} + \vec{c} = 0$ (Given)

$\vec{a}.\left(\vec{a} + \vec{b} + \vec{c}\right) = 0$

$\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$

$\left|\vec{a}\right|^2 + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$

$1 + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$

$\vec{a}.\vec{b} + \vec{a}.\vec{c} = -1$ ...(i)

Similarly,

$\vec{b}.\vec{a} + \vec{b}.\vec{c} = -1$ ...(ii)

again

$\vec{c}.\vec{a} + \vec{c}.\vec{b} = -1$ ...(iii)

adding (i), (ii) and (iii)

$2\left(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}\right) = -3\left[\because \vec{a}.\vec{b} = \vec{b}.\vec{a}\right]$

$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\frac{3}{2}$

34. Given that A = {2, 3, 4}, B = {2, 5, 6, 7}

1. Let $f: A \to B$ defined by
   f = {(x, y) : y = x + 3}

i.e. f = {(2, 5), (3, -6), (4, 7)} f = {(2, 5), (3, 6), (4, 7)} which is an injective mapping.

2. Let $g: A \to B$ denote a mapping such that g = {(2, 2), (3, 5), (4, 5) which is not an injective mapping.
3. Let $h: B \to A$ denote a mapping such that h = {(2, 2), (5, 3), (6, 4), (7, 4)} which is a mapping from B to A.

OR

i. Let $(a_1\ b_1)$ and $(a_2, b_2) \in A \times B$ such that

$f(a_1, b_1) = f(a_2, b_2)$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

$\Rightarrow a_1 = a_2$ and $b_1 = b_2$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

Therefore, f is injective.

ii. Let (b, a) be an arbitrary

Element of $B \times A$. then $b \in B$ and $a \in A$

$\Rightarrow (a, b)\ ) \in (A \times B)$

Thus for all $(b, a) \in B \times A$ their exists $(a, b)\ ) \in (A \times B)$ such that

f (a, b) = (b, a)

So $f: A \times B \to B \times A$

is an onto function.

Hence f is bijective.

35. $$I' = \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\,dx$$

$$\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \frac{\pi}{2}$$

$$\cos^{-1}\sqrt{x} = \frac{\pi}{2} - \sin^{-1}\sqrt{x}$$

$$I' = \int \frac{\sin^{-1}\sqrt{x} - \left(\frac{\pi}{2} - \sin^{-1}\sqrt{x}\right)}{\pi/2}\,dx$$

$$= \int \frac{2\sin^{-1}\sqrt{x} - \frac{\pi}{2}}{\pi/2}\,dx$$

$$= \frac{4}{\pi}\int \sin^{-1}\sqrt{x}dx - \int 1dx$$

$$= \frac{4}{\pi}I - x + c \text{ ...(i)}$$

$$I = \int \sin^{-1}\sqrt{x}dx$$

Put $\sin^{-1}\sqrt{x} = t$

$\sin t = \sqrt{x}$

$\sin^2 t = x$

$2\sin t\cos t = dx$

$I = \int t.2\sin t\cos t\, dt$

$= \int t.\sin 2t\, dt$

$\quad I \quad II$

$$= -t\frac{\cos 2t}{2} - \int 1.\left(-\frac{\cos 2t}{2}\right)dt$$

$$= \frac{-t\cos 2t}{2} + \frac{1}{2}\frac{\sin 2t}{2} + c$$

$$= \frac{-t(1-2\sin^2 t)}{2} + \frac{1}{4}2\sin t\cos t + c$$

$$= \frac{-t(1-2\sin^2 t)}{2} + \frac{1}{2}\sin t\sqrt{1-\sin^2 t} + c$$

$$= \frac{-\sin^{-1}\sqrt{x}(1-2x)}{2} + \frac{1}{2}.\sqrt{x}\sqrt{1-x} + c$$

$$= \frac{\sin^{-1}\sqrt{x}(1-2x)}{2} + \frac{1}{2}.\sqrt{x}\sqrt{1-x} + c$$

$$= \frac{\sin^{-1}\sqrt{x}(2x-1)}{2} + \frac{1}{2}.\sqrt{x-x^2} + c$$

From (i),

$$I' = \int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}dx = \frac{4}{\pi}\left(\frac{\sin^{-1}\sqrt{x}(2x-1)}{2} + \frac{1}{2}\sqrt{x-x^2}\right) - x + c$$

Section E

36. **Read the text carefully and answer the questions:**

Naina is creative she wants to prepare a sweet box for Diwali at home. She took a square piece of cardboard of side 18 cm which is to be made into an open box, by cutting a square from each corner and folding up the flaps to form the box. She wants to cover the top of the box with some decorative paper. Naina is interested in maximizing the volume of the box.

(i) Let the side of square to be cut off be 'x' cm. then, the length and the breadth of the box will be (18 - 2x) cm each and the height of the box is 'x' cm.

The volume V(x) of the box is given by $V(x) = x(18 - x)^2$

(ii) $V(x) = x(18 - 2x)^2$

$$\frac{dV(x)}{dx} = (18 - 2x)^2 - 4x(18 - 2x)$$

For maxima or minima $= \frac{dV(x)}{dx} = 0$

$\Rightarrow (18 - 2x)[18 - 2x - 4x] = 0$

$\Rightarrow x = 9$ or $x = 3$

$\Rightarrow x =$ not possible

$\Rightarrow x = 3$ cm

The side of the square to be cut off so that the volume of the box is maximum is x = 3 cm

(iii) $\frac{dV(x)}{dx} = (18 - 2x)(18 - 6x)$

$$\frac{d^2V(x)}{dx^2} = (18 - 6x)(-2) + (18 - 2x)(-6)$$

$$\Rightarrow \frac{d^2V(x)}{dx^2} = -12[3 - x + 9 - x] = -24(6 - x)$$

$$\Rightarrow \left.\frac{d^2V(x)}{dx^2}\right]_{x=3} = -72 < 0$$

$\Rightarrow$ volume is maximum at x = 3

OR

$V(x) = x(18 - 2x)^2$

When x = 3

$V(3) = 3(18 - 2 \times 3)^2$

$\Rightarrow$ Volume $= 3 \times 12 \times 12 = 432$ cm^3

37. **Read the text carefully and answer the questions:**

The nut and bolt manufacturing business has gained popularity due to the rapid Industrialization and introduction of the Capital-Intensive Techniques in the Industries that are used as the Industrial fasteners to connect various machines and structures. Mr. Suresh is in Manufacturing business of Nuts and bolts. He produces three types of bolts, x, y, and z which he sells in two markets. Annual sales (in ₹) indicated below:

Markets	Products		
	x	y	z
I	10000	2000	18000
II	6000	20000	8000

(i) Let A be the 2 × 3 matrix representing the annual sales of products in two markets.

$$A = \begin{array}{c c} & \begin{array}{ccc} x & y & z \end{array} \\ & \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{array}{l} \textit{Market I} \\ \textit{Market II} \end{array} \end{array}$$

Now, revenue = sale price × number of items sold

$$AB = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2.5 \\ 1.5 \\ 1 \end{bmatrix}$$

$$\Rightarrow AB = \begin{bmatrix} 25000 + 3000 + 18000 \\ 15000 + 30000 + 8000 \end{bmatrix} = \begin{bmatrix} 46000 \\ 53000 \end{bmatrix}$$

Therefore, the revenue collected from Market I = ₹46000 and the revenue collected from Market II = ₹53000.

(ii) Let C be the column matrix representing cost price of each unit of products x, y, z.

$$C = \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$$

$$AC = \begin{bmatrix} 10000 & 2000 & 18000 \\ 6000 & 20000 & 8000 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0.5 \end{bmatrix}$$

$$\Rightarrow AC = \begin{bmatrix} 20000 + 2000 + 9000 \\ 12000 + 20000 + 4000 \end{bmatrix} = \begin{bmatrix} 31000 \\ 36000 \end{bmatrix}$$

Cost price in Market I is ₹31000 and in Market is ₹36000.

(iii) Now, Profit matrix = Revenue matrix - Cost matrix

$\Rightarrow$ AB - AC

$$\Rightarrow \begin{bmatrix} 46000 \\ 53000 \end{bmatrix} - \begin{bmatrix} 31000 \\ 36000 \end{bmatrix} = \begin{bmatrix} 15000 \\ 17000 \end{bmatrix}$$

Therefore, the gross profit from both the markets = ₹15000 + ₹17000 = ₹32000

OR

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$\Rightarrow A^2 = I$

38. **Read the text carefully and answer the questions:**

Mr. Ajay is taking up subjects of mathematics, physics, and chemistry in the examination. His probabilities of getting a grade A in these subjects are 0.2, 0.3, and 0.5 respectively.

(i) P(Grade A in Maths) = P(M) = 0.2

P(Grade A in Physics) = P(P) = 0.3

P(Grade A in Chemistry) = P(C) = 0.5

P(Grade A in all subjects) = $P(M \cap P \cap C) = P(M) \cdot P(P) \cdot P(C)$

P(Grade A in all subjects) = $0.2 \times 0.3 \times 0.5 = 0.03$

(ii) P(Grade A in Maths) = P(M) = 0.2

P(Grade A in Physics) = P(P) = 0.3

P(Grade A in Chemistry) = P(C) = 0.5

P(Grade A in no subjects) = $P(\bar{M} \cap \bar{P} \cap \bar{C}) = P(\bar{M}) \cdot P(\bar{P}) \cdot P(\bar{C})$

P(Grade A in no subjects) = $0.8 \times 0.7 \times 0.5 = 0.280$